## Quantum ordering structures in inhomogeneous systems --- Super-solid state in the interface region -----

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It is an interesting problem to find in what condition the super-solid state appears.[1-5] So far, the condition has been studied in uniform system. However, by the recent development of nanoscale design of material and also of the technique of optical lattice, we may control inhomogeneous structure of the lattice. Here we will study a system where the system parameters change in space and study the local ordering patterns. In particular, we explore the possibility to find the coexistence of the spatial density order (DLRO) and the super-fluidity (ODLRO). We study the following spin model (Matsubara-Matsuda model) :

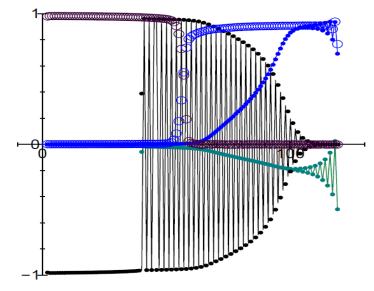
$$H = \sum_{\langle ij \rangle} \left( J_{xy}(i,j) \left( S_i^{x} S_j^{x} + S_i^{y} S_j^{y} \right) + J_{z}(i,j) S_i^{z} S_j^{z} \right) + \sum_{\langle ik \rangle} J_{z}'(i,k) S_i^{z} S_k^{z} - \sum_i H(i) S_i^{z},$$

where  $\langle ij \rangle$  denotes the nearest neighbor pairs, and  $\langle ik \rangle$  denote the next nearest neighbor pairs on the square lattice. Here the parameters change as follows:

 $J_{xy}(i, j) = 2i/120, J_z(i, j) = 2, J_z'(i, k) = 0.5 + 1.0i/120$ , and H(i, j) = -5i/120.

Here, the lattice length is 120. The black circles (left) denote the staggered magnetization of the z-component in j-direction which corresponds to the DRLO of solid. The blue circles (right) denote the magnetization in the xy plane, which corresponds to the super fluidity. We find overlap region between them. This observation suggests that a kind of super-solid behavior may be observed in the interface region. Here, we study the classical system, but we will report also for the S=1/2 case.

We also studied the dependence of the degree of the frustration by choosing J'. The data for  $J_{z'}(i,k) = 0.5 + 1.5i/120$  are plotted by dots: The green dots (below the axis) denote the uniform magnetization which corresponds the density of particles. In this case we find two types of zigzag structures, which correspond to the density alternation in the j and i directions. We discuss the meaning of these configurations.



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