Ring Exchanges in Solid ^{3}He

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Until today, there are few examples where many-body interactions are essentially important in nature. One fairly well established example is the many-body exchanges in solid helium. Solid ^{3}He, a fermionic quantum crystal with either bcc or hcp structure, is an ideal system where one can study these effects quantitatively by measuring its nuclear magnetism. This is because the atomic exchanges give rise to magnetic interactions among the nuclear spins through the Pauli principle. The light mass of He atom results in non-negligible probabilities of quantum tunneling between neighboring atoms due to the Heisenberg uncertain principle. The hard-core nature of the interatomic interaction requires that even the two-body exchange processes are essentially of many body, and that the exchange paths are restricted along rings (or loops). Thouless [1] was the first to point out the importance of such ring exchanges in solid ^{3}He, and he gave the effective spin Hamiltonian up to the four-spin exchange.

Exchange frequencies (J_P) in solid ^{3}He are of the order of millikelvin. Thus the nuclear magnetism should be explored at temperatures comparable to 1 mK or lower. There are many experimental facts that are explained only by considering the ring exchanges, for example, the first order transition to the U2D2 antiferromagnetic ordered phase in low magnetic fields, the existence of the canted-antiferromagnetic ordered phase with ferromagnetic tendency in high fields, etc [2]. A remarkable feature of this system is that one can compare J_P determined experimentally with those calculated by the path integral Monte Carlo [3] and WKB [4] methods. So far, the agreement between the experiments and those first-principles calculations is satisfactory. The current consensus is that the two-, three- and four-spin exchanges are competing each other roughly in the same order (see Fig. 1) and that the inclusion of the six-spin ring exchanges improve the agreement further [5]. However, it should be noted that we still don't know exactly where one can truncate the series of higher ring exchanges.

In this lecture, I will outline basic, historical and current knowledge of the ring exchanges mainly in three dimensional solid ^{3}He.

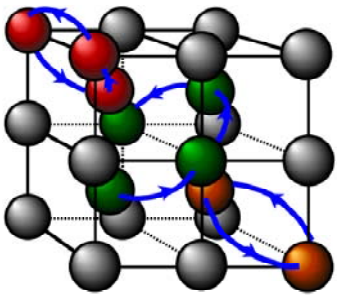

Fig.1: Two-, three-, and four-body exchanges in bcc solid ^{3}He.

[1] D.J. Thouless, Proc. Phys. Soc. London **86**, 893 (1965); **86**, 905 (1965).
[2] M. Roger, J.H. Hetherington, and J.M. Delrieu, Rev. Mod. Phys. **55**, 1 (1983).
[3] D.M. Ceperley and G. Jacucci, Phys. Rev. Lett. **58**, 1648 (1987).
[4] M. Roger, Phys. Rev. B **30**, 6432 (1984).
[5] H. Fukuyama *et al*., cond-mat/0505177.